# SIMPLE OLS REGRESSION, PART II: GOODNESS OF FIT 

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Last Update: February 14, 2016

## Example

## Variables Entered/Removed ${ }^{\text {a }}$

| Model | Variables <br> Entered | Variables <br> Removed | Method |
| :--- | :---: | :---: | :---: |
| 1 | Poverty Rate $^{\text {b }}$ |  | Enter |

a. Dependent Variable: Log of Violent Crime

Rate
b. All requested variables entered.

Model Summary

| Model | R | R Square | Adjusted R <br> Square | Std. Error of <br> the Estimate |
| :--- | :--- | ---: | ---: | ---: |
| 1 | $.391^{\mathrm{a}}$ | .152 | .135 | .36130 |

a. Predictors: (Constant), Poverty Rate

a. Dependent Variable: Log of Violent Crime Rate
b. Predictors: (Constant), Poverty Rate

Coefficients ${ }^{\text {a }}$

| Model |  | Unstandardized Coefficients |  | Standardized <br> Coefficients <br> Beta | t | Sig. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B | Std. Error |  |  |  |
| 1 | (Constant) | 5.097 | . 243 |  | 20.981 | . 000 |
|  | Poverty Rate | . 046 | . 016 | . 391 | 2.939 | . 005 |

## Scatterplot of the Relationship



## Model Summary and ANOVA Tables

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a. Predictors: (Constant), Poverty Rate

| $\text { ANOVA }{ }^{\text {a }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Model | Sum of Squares | df | Mean Square | F | Sig. |
| 1 Regression | 1.127 | 1 | 1.127 | 8.637 | $.005^{\text {b }}$ |
| Residual | 6.266 | 48 | .131 |  |  |
| Total | 7.393 | 49 |  |  |  |

a. Dependent Variable. Log of viotent Crime Rate
b. Predictors: (Constant), Poverty Rate

## Explaining Sums of Squares

$$
\bar{y}=3
$$

## Explaining Sums of Squares

$$
\bar{y}=3
$$

$$
(1,1)
$$

$$
\mathrm{y}=1
$$

## Explaining Sums of Squares



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## Sum of Squares

Model Summary

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| :--- | :--- | ---: | ---: | ---: |
| 1 | $.391^{\mathrm{a}}$ | .152 | .135 | .36130 | $R^{2}=\frac{\text { regression ss }}{\text { total } s s}$

a. Predictors: (Constant), Poverty Rate

ANOVA ${ }^{\text {a }}$

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## Adjusted R ${ }^{2}$

Model Summary

| Model | R | R Square | Adjusted R <br> Square | Std. Error of <br> the Estimate |
| :--- | :--- | ---: | ---: | ---: |
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a. Predictors: (Constant), Poverty Rate

$$
\text { Adj. } R^{2}=1-(1-R 2) \frac{n-1}{n-p-1}=R^{2}-\left(1-R^{2}\right) \frac{p}{n-p-1}
$$

| Model |  | Sum of <br> Squares | df | Mean Square | F | Sig. |
| :--- | :--- | ---: | ---: | ---: | :---: | :---: |
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## Statistical Significance of the Model

Model Summary

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ANOVA ${ }^{\text {a }}$

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| :--- | :--- | ---: | ---: | ---: | ---: | :---: |
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## Degrees of Freedom

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| Model | R | R Square | Adjusted R <br> Square | Std. Error of <br> the Estimate |
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## Mean Square (Mean Square Error)

Model Summary

| Model | R | R Square | Adjusted R <br> Square | Std. Error of <br> the Estimate |
| :--- | :--- | ---: | ---: | ---: |
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a. Predictors: (Constant), Poverty Rate

| ANOVA ${ }^{\text {a }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Model | Sum of Squares | df | Mean Square | F | Sig. |
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| :--- | :--- | ---: | ---: | ---: |
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$$
F=\frac{\text { Regression Mean Square }}{\text { Residual Mean Square }}
$$

ANOVA ${ }^{\text {a }}$

| Model |  | Sum of <br> Squares | df | Mean Square | F | Sig. |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
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## F-Distribution



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## Simple OLS Regression

- Simple OLS regression is used when both dependent and independent variables are numeric.
- From beta and the standardized beta, we learn direction and magnitude. The beta coefficients can be tested for their statistical significance.
- We can use the sum of squares to estimate the fit of the model ( $\mathrm{R}^{2}$ ). The fit of the model also has statistical significance.


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