SIMPLE OLS REGRESSION, PART I: THE EQUATION OF A LINE

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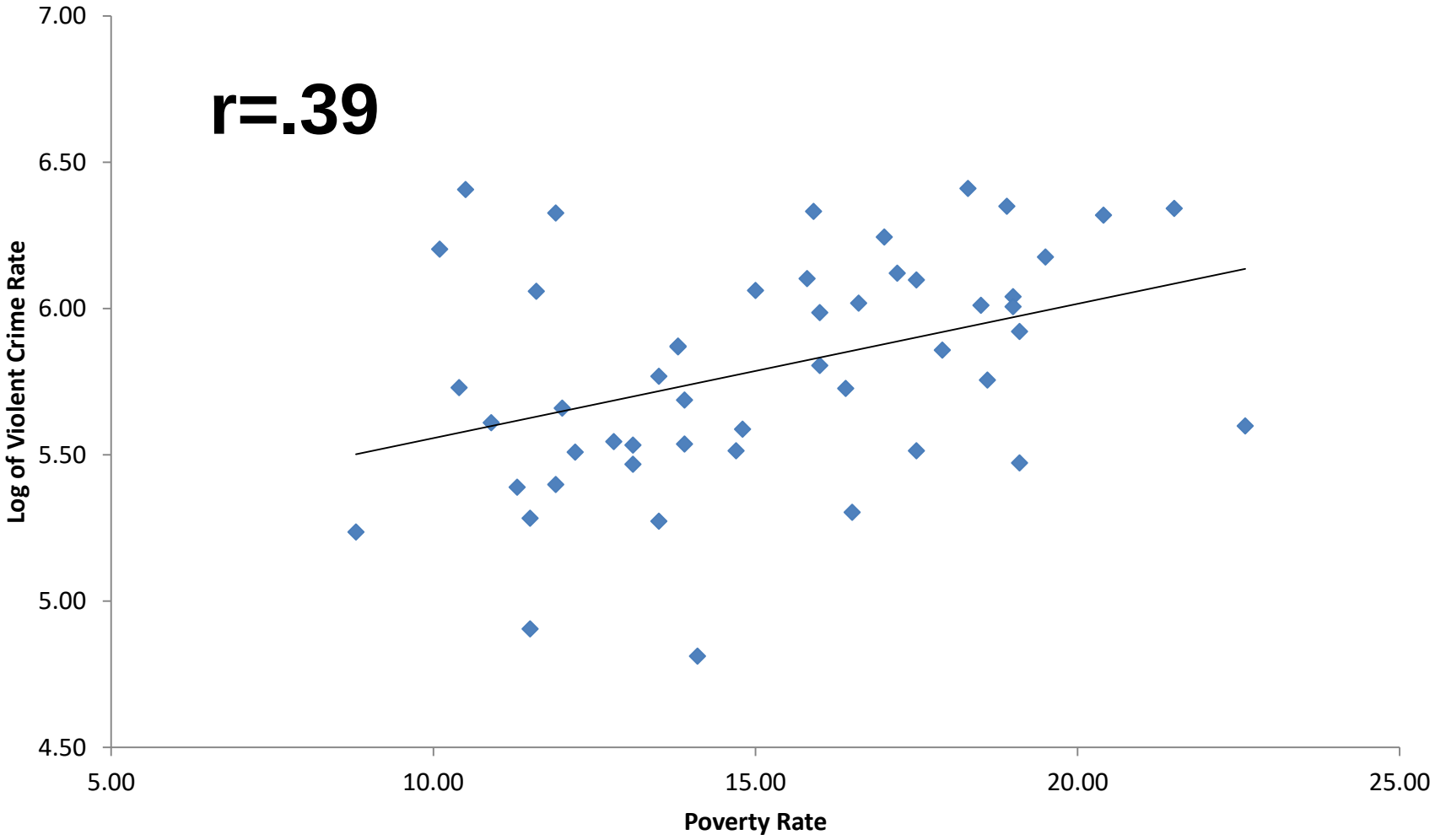
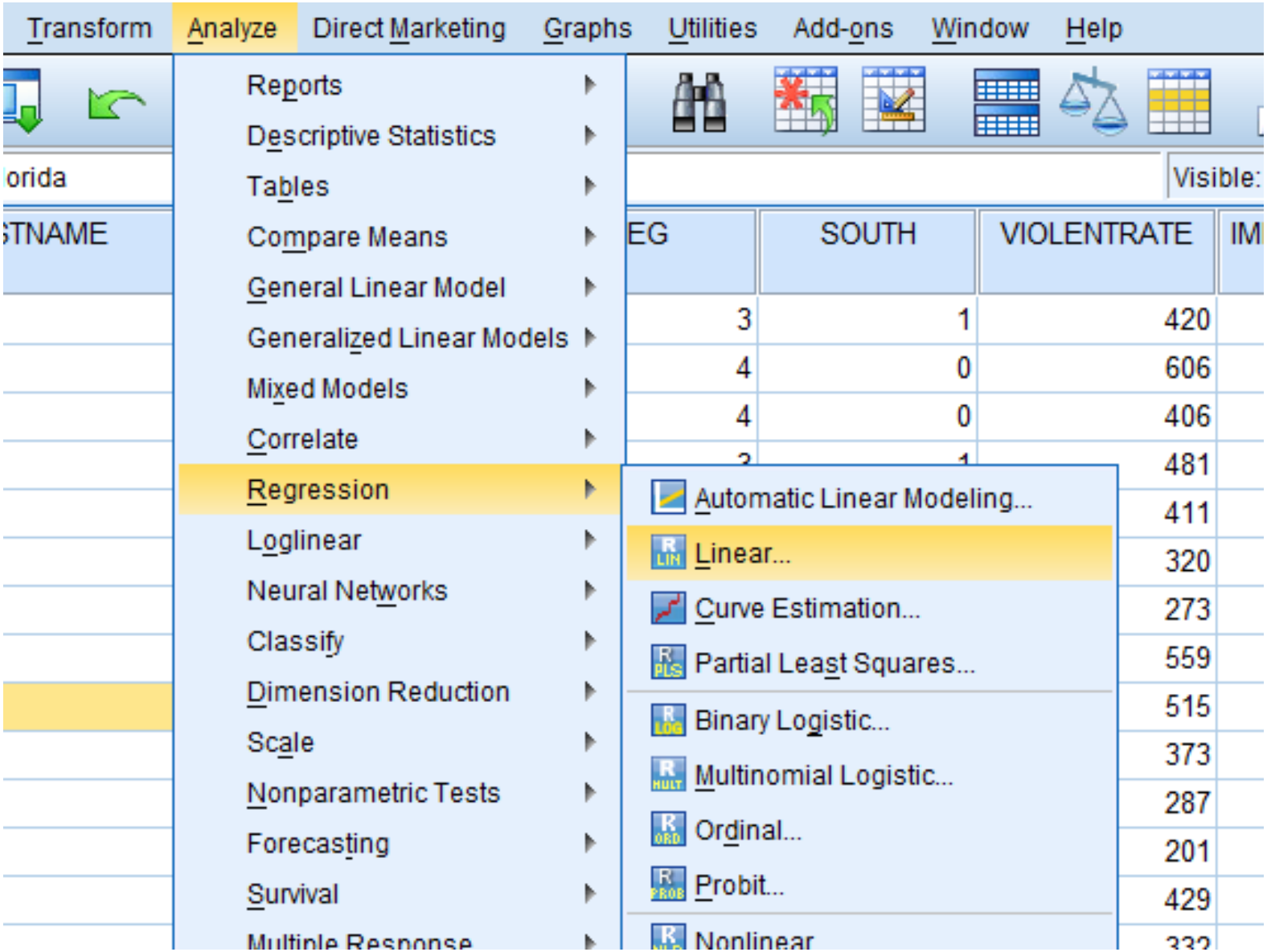
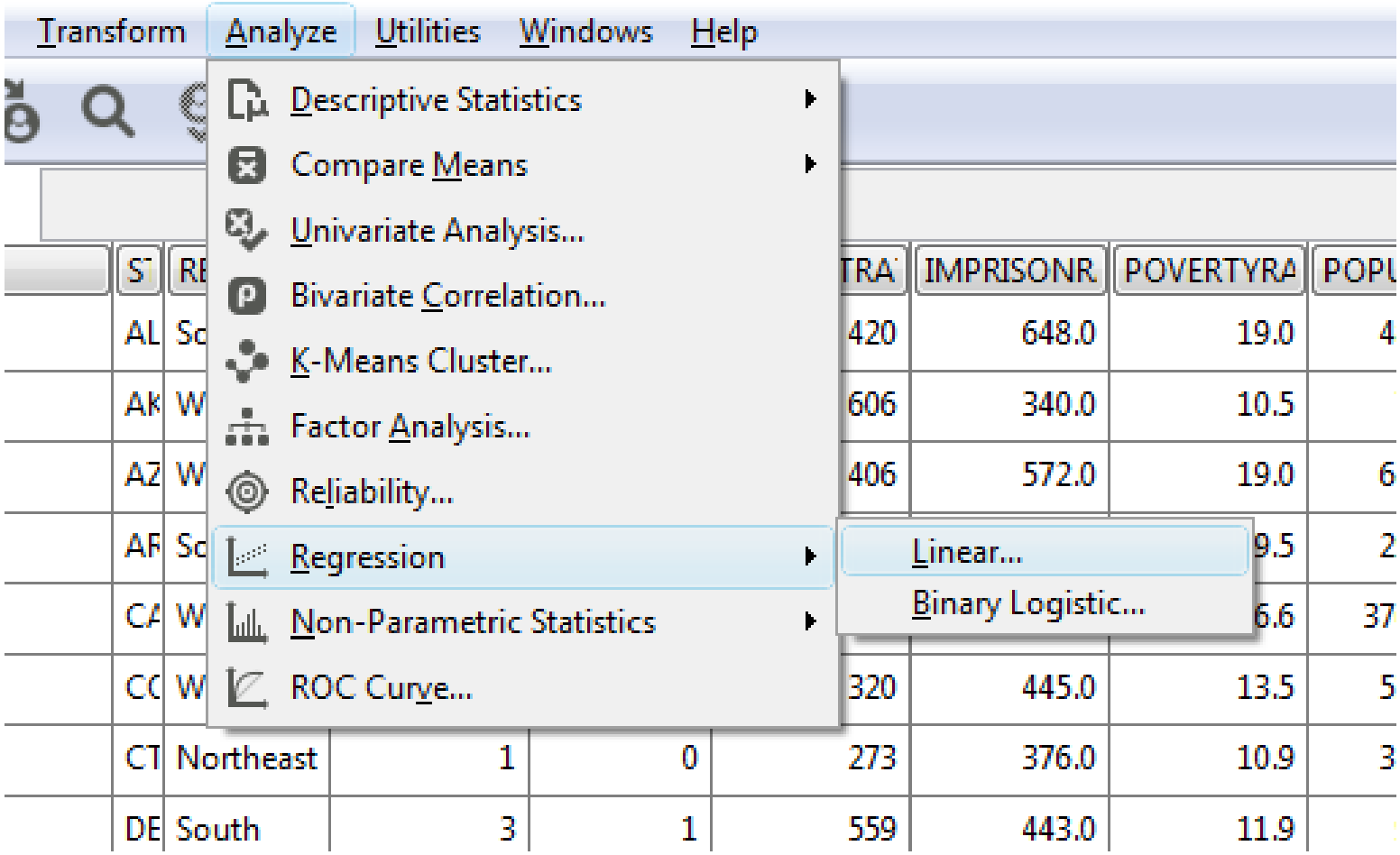
Example


r=.39
7.00
6.50
6.00
5.50
5.00
4.50
Log of Violent Crime Rate
5.00
10.00
15.00
20.00
25.00
Poverty Rate

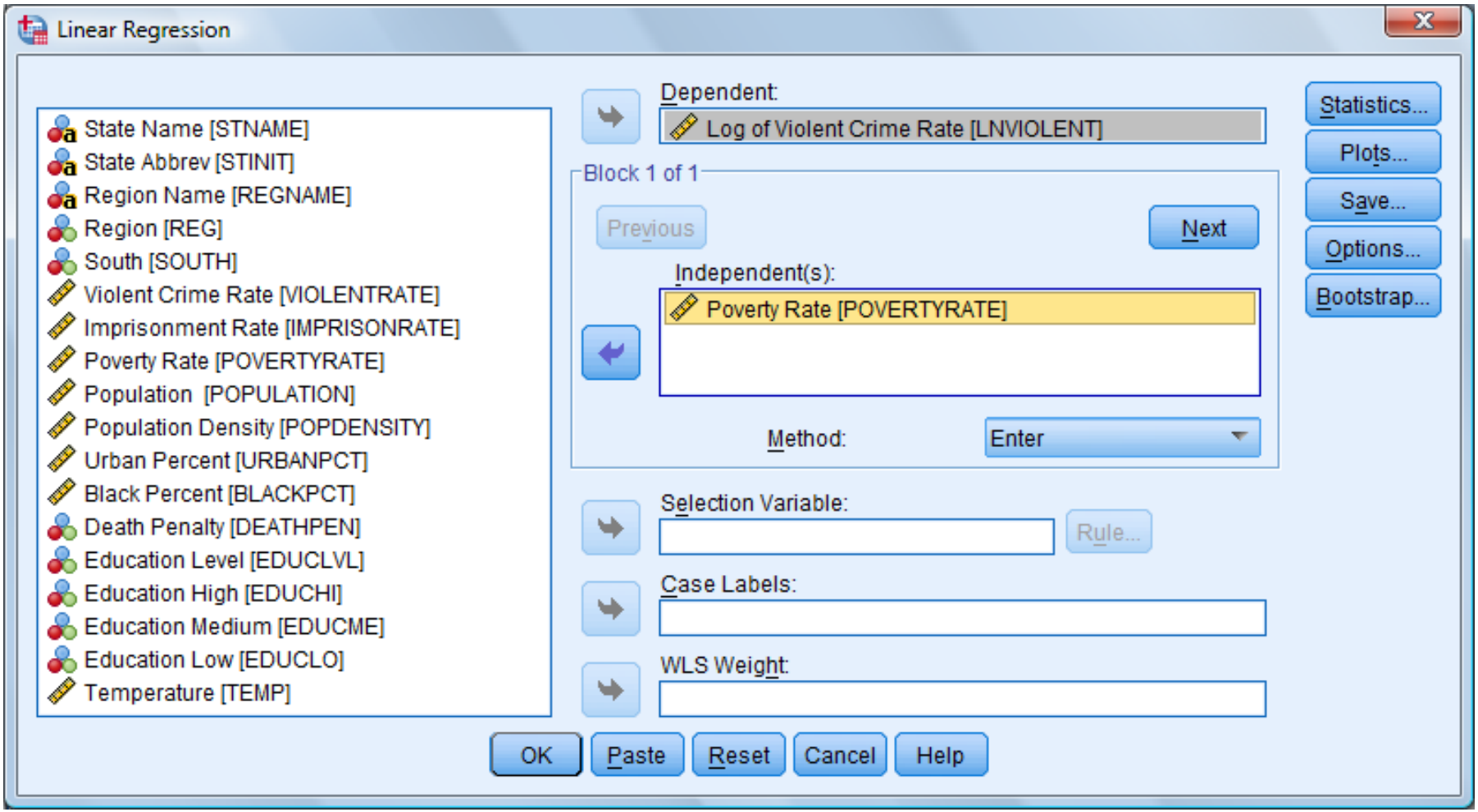
Analyze > Regression > Linear



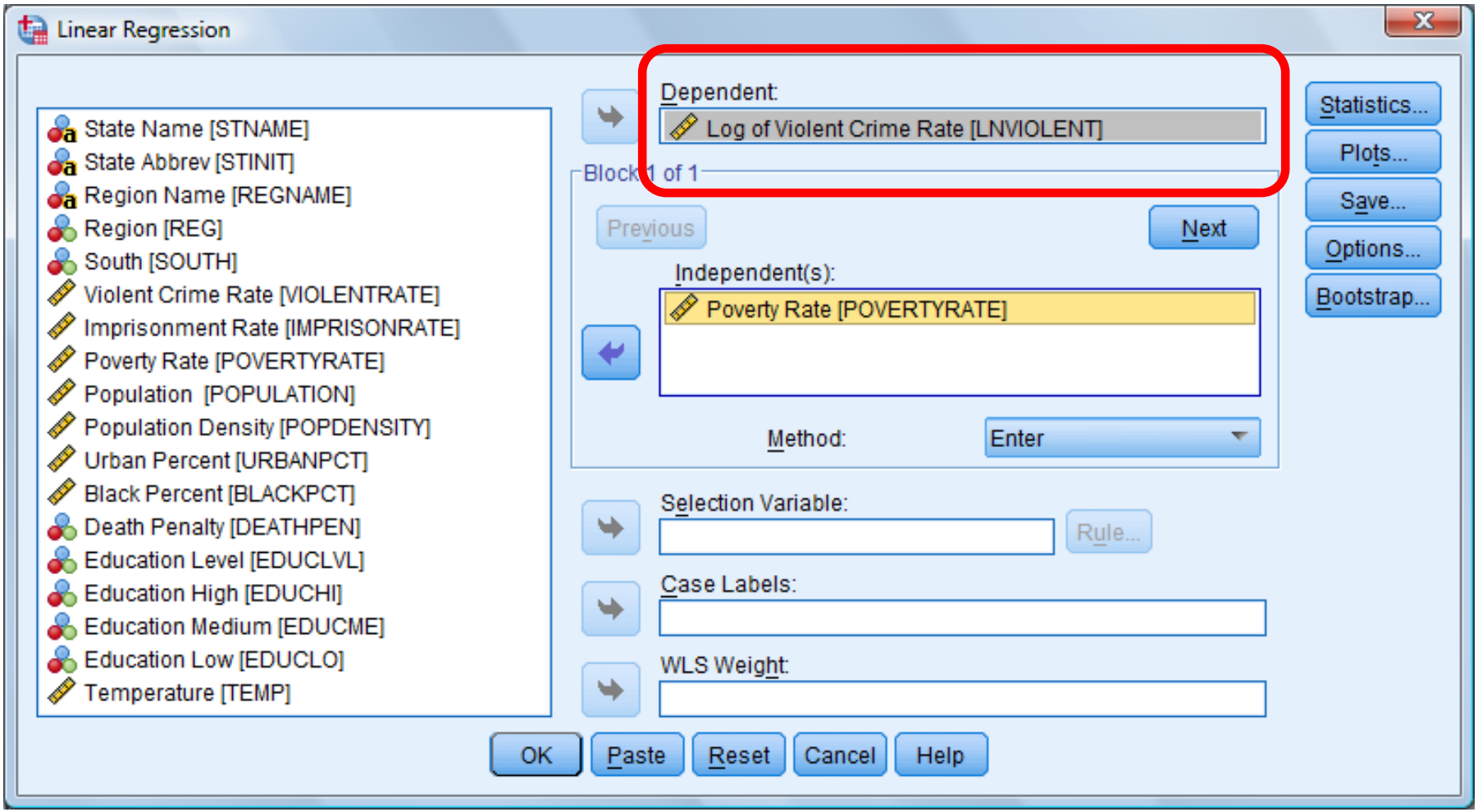
Analyze > Regression > Linear



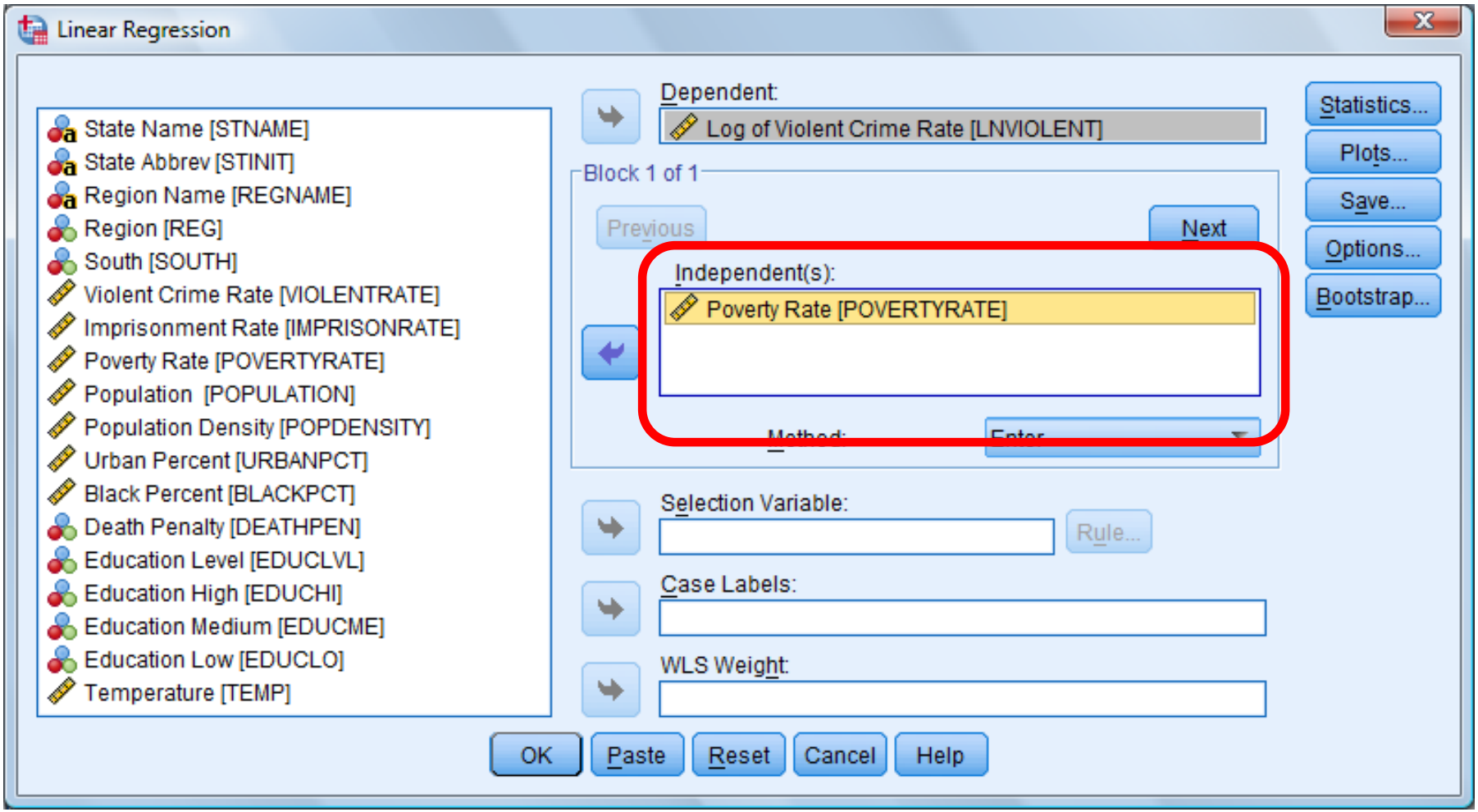
Command Dialog Box



Command Dialog Box



Command Dialog Box



Output

Variables Entered/Removed[a]

Model	Variables Entered	Variables Removed	Method
1	Poverty Rate[b]	.	Enter

a. Dependent Variable: Log of Violent Crime Rate

b. All requested variables entered.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.391[a]	.152	.135	.36130

a. Predictors: (Constant), Poverty Rate

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1.127	1	1.127	8.637	.005[b]
	Residual	6.266	48	.131		
	Total	7.393	49			

a. Dependent Variable: Log of Violent Crime Rate

b. Predictors: (Constant), Poverty Rate

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
	Poverty Rate	.046	.016	.391	2.939	.005

Parameters of the Line

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
	Poverty Rate	.046	.016	.391	2.939	.005

a. Dependent Variable: Log of Violent Crime Rate

Parameters of the Line: B_o

b_o

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
	Poverty Rate	.046	.016	.391	2.939	.005

a. Dependent Variable: Log of Violent Crime Rate

Parameters of the Line: B_1

b_1

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
	Poverty Rate	.046	.016	.391	2.939	.005

a. Dependent Variable: Log of Violent Crime Rate

The Equation of the Line

Coefficients[a]

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a. Dependent Variable: Log of Violent Crime Rate

The Equation of the Line

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a. Dependent Variable: Log of Violent Crime Rate

$$\hat{y} = b_o + b_1x_1 = 5.10 + .05(\text{poverty rate})$$

The Equation of the Line

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
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1	(Constant)	5.097	.243		20.981	.000
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a. Dependent Variable: Log of Violent Crime Rate

$$\hat{y} = b_o + b_1 x_1 = 5.10 + .05(\text{poverty rate})$$

$$b_1 = \frac{\text{covariance of x and y}}{\text{variance of x}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$$

The Equation of the Line

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
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a. Dependent Variable: Log of Violent Crime Rate

$$\hat{y} = b_o + b_1 x_1 = 5.10 + .05(\text{poverty rate})$$

$$b_1 = \frac{\text{covariance of x and y}}{\text{variance of x}} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = r_{xy}\frac{s_y}{s_x}$$

What the Slope Tells Us

- Sign of b_1 is the direction of the relationship.
- The magnitude of b_1 is unstandardized
- In this analysis
  - The relationship is positive
  - Magnitude: The log of the violent crime rate increases .05 for every percent increase in the poverty rate

Standard Error of the Beta Coefficient

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
	Poverty Rate	.046	.016	.391	2.939	.005

a. Dependent Variable: Log of Violent Crime Rate

Standardized Beta

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
	Poverty Rate	.046	.016	.391	2.939	.005

a. Dependent Variable: Log of Violent Crime Rate

Statistical Significance

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
	Poverty Rate	.046	.016	.391	2.939	.005

a. Dependent Variable: Log of Violent Crime Rate

Null Hypotheses:

$b_o = 0$

$b_1 = 0$

t

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.097	.243		20.981	.000
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a. Dependent Variable: Log of Violent Crime Rate

Elements of An Inferential Statistic

Elements in the parameter of a line are associated with B1

- Direction: sign
- Unstandardized magnitude: beta (slope)
- Standardized magnitude: standardized beta
- Statistical significance

Constant usually has no substantive bearing on the relationship

Example results

- The relationship is positive
- The relationship is moderate (standardized magnitude: b_1=.39)
- Statistically significant
- The log of violent crime rate increases .05 for each percent increase in the poverty rate (unstandardized beta: b_o=.05)